RESISTANCE OF A CYLINDER BUNDLE IN A LOW-DENSITY GAS FLOW

E. N. Inozemtseva, Yu. A. Koshmarov,
S. B. Svirshchevskii, and D. S. Strizhenov

UDC 533.6.011.8

The apparatus of [1] has been used in examining the resistance of cylinders transverse to an air flow in the transition range. The working part consisted of a channel of cross section $l_1 \times l_2 = 22 \times 60$ mm (l_1 and l_2 are the width and height) containing packets of cylinders built up from wires of various diameters (d = 0.5, 0.6, 1, 1.45, 1.8, 2 mm), which were mounted on two copper plates. The rectangular channel was cut in a baffle made of Lucite of thickness 25 mm, which separated the working part into two vacuum chambers. The baffle bearing the packet was installed in the evacuated tube in such a way that the velocity vector for the incident flow was perpendicular to the cylinders. Preliminary experiments and various pressures showed that the effects of the walls on the bundle resistance can be neglected if a transverse series in a packet contains more than six wires.

The studies were made with six cylinder packets. Each packet consisted of ten rows (z = 10), with each row containing ten cylinders ($\kappa = 10$). In five of the packets, the wires were arranged in chessboard order on an equilateral triangle. The relative steps in the packets varied over the following ranges: $\sigma_1 = S_1/d = 1.15\text{–}3.83$, $\sigma_2 = S_2/d = 1\text{–}3.33$, where $S_1 = 2.3$ mm and $S_2 = 2$ mm are the transverse and longitudinal steps in relation to the flow. In the sixth packet, the wires (d = 0.5 mm) were arranged in an isosceles right-angle triangle ($\sigma_1 = 1.6$, $\sigma_2 = 0.8$).

The resistance was determined from the difference in the static pressures Δp measured in the chambers on each side. The pressure p in the flow and the pressure difference Δp were measured by the method of [1]. The standard deviation in pressure and pressure-difference measurement was ±2%. The gas flow rate G was measured by a volumetric method at constant pressure. The standard deviation in measuring flow rates in the range $(0.01\text{–}5)\cdot 10^{-4}$ N/sec was not more than ±7%.

The chess-array bundles were examined under isothermal conditions (T = 290°K) at p = 6.67–2940 Pa for Knudsen numbers $\mathrm{Kn_h} = 1.27\sqrt{k}M/\mathrm{Re_h} = 0.01\text{–}7$ and Reynolds numbers $\mathrm{Re_h} = \rho u d_h/\mu = 0.013\text{–}26$, as calculated from the hydraulic bundle diameter $d_h = d[4\sigma_2(\sigma_1 - 1)]/\pi$ [1], with the Mach number M = 0.05–0.3 and a relative bundle step $t = 2\sigma_1 = 2.3\text{–}7.66$. The gas density ρ was calculated from the average pressure in the bundle. The characteristic velocity was taken as the velocity in the minimal cross section of the packet $u = G/g\rho f_1$, where $f_1 = \kappa(S_1 - d)l_2$, g is the acceleration due to gravity, and $k = c_p/c_V$ is the adiabatic parameter.

The experimental data (Fig. 1) were plotted as

$$\mathrm{Eu} = f(\mathrm{Kn_h})\quad \mathrm{Eu} = \Delta p/\rho u^2 z,$$

where Eu is the Euler number referred to a single row in the bundle.

In Fig. 1, the points 1 relate to bundles of diameter d = 2 mm with Mach numbers of M = 0.107–0.048; 2) d = 1.8 mm, M = 0.157–0.057; 3) d = 1.45 mm, M = 0.109–0.069; 4) d = 1 mm, M = 0.243–0.165; 5) d = 0.6 mm, M = 0.307–0.226; 6) d = 0.5 mm, M = 0.119–0.061. The standard deviation in the determination was ±10%.

Figure 1 shows that in the transition region the pressure has different effects on the resistance in accordance with the step. The experimental data separate into distinct trends as $\mathrm{Kn_h}$ increases, and the resistance tends to the asymptotic free-molecular limit. We also show experimental points 7 (d = 16–24 mm, M = 0.03–0.06) on the resistance of tube bundles at low Reynolds numbers. The solid lines show the following [1]:

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 44-48, March-April, 1984. Original article submitted March 22, 1983.

0021-8944/84/2502-0205$08.50

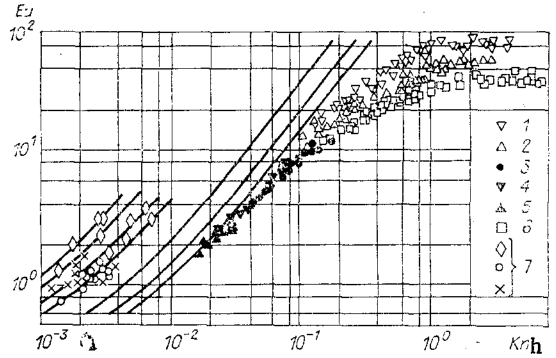

Fig. 1

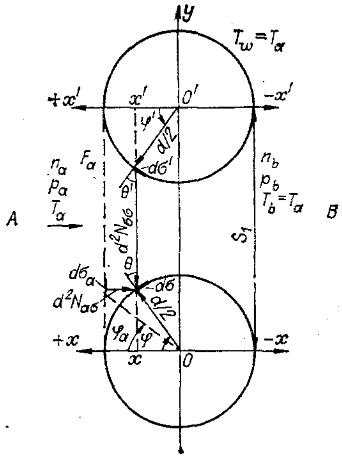

Fig. 2

$$\mathrm{Eu}_c = 21\pi\,\mathrm{Re}_h^{-1}\left(1 - 0{,}324\,\mathrm{Re}_h^{0,5} + 0{,}149\,\mathrm{Re}_h^{2/3}\right), \tag{1*}$$

which describes the resistance of a chessboard tube bundle in the continuum region for various M.

To establish the flow pattern in the free-molecular region, we considered isothermal subsonic flow (M << 1) between circular cylinders (Fig. 2). The flow in the space between the tubes was considered as the superposition of two molecular flows: from chambers A and B [2]. It was assumed that the gas ahead of the cylinders and behind them was in equilibrium states with densities n_a and n_b, temperatures $T = T_a = T_b$, and pressures $p_a > p_b$. Diffuse reflection from the walls of the cylinders was assumed. The balance between the incident and reflected particles may be written on the basis of interference for an element $d\sigma$ in the cylindrical surface:

$$dN_{\sigma r} = dN_{\sigma\sigma} + dN_{a\sigma}, \tag{2}$$

where $dN_{\sigma r} = n_r(x)\sqrt{\frac{R_0 T}{2\pi}}\,d\sigma$ is the particle flux from the element $d\sigma$; $n_r(x)$, density of the particles reflected from the cylindrical wall; R_0, gas constant; $dN_{\sigma\sigma}$, flux to the element $d\sigma$ of particles reflected from the surface of another cylinder; and $dN_{a\sigma}$, flux of particles from chamber A to element $d\sigma$.

We substitute the expressions for the molecular flows into (2) and transform it to get the following integral Fredholm equation of the second kind for the reflected-particle density, where the definitive parameter is the relative bundle step $t = 2S_1/d$:

$$w(\varphi) = \frac{1}{2}\int_0^{\pi} w(\varphi')\,K(\varphi,\varphi')\,d\varphi' + Q(\varphi), \tag{3}$$

where $w(\varphi) = n_r(\varphi)/n_a$, $0 \leqslant \varphi \leqslant \pi$, $t \geqslant 2$, and

$$K(\varphi,\varphi') = \begin{cases} \dfrac{1}{2}\,\dfrac{a(\varphi,\varphi')\,b(\varphi,\varphi')}{[(\cos\varphi - \cos\varphi')^2 + (t - \sin\varphi - \sin\varphi')^2]^{3/2}} & \text{for} \quad a > 0 \wedge b > 0, \\ 0 & \text{otherwise,} \end{cases}$$

$$a(\varphi,\varphi') = t\sin\varphi + \cos(\varphi + \varphi') - 1,$$

$$b(\varphi,\varphi') = t\sin\varphi' + \cos(\varphi + \varphi') - 1,$$

$$Q(\varphi) = \begin{cases} \dfrac{1}{2}\left[1 + \dfrac{1 - t\sin\varphi + t\cos\varphi\sqrt{t^2 - 2t\sin\varphi}}{1 + t^2 - 2t\sin\varphi}\right] & \text{for} \quad 0 \leqslant \varphi \leqslant \pi - \arcsin\dfrac{2}{t}, \\ 0 & \text{for} \quad \pi - \arcsin\dfrac{2}{t} \leqslant \varphi \leqslant \pi. \end{cases}$$

*Formula (1) coincides numerically with the formula given in [1].

The norm of the operator on the right in (3) is

$$\left\| \int_0^\pi K(\varphi, \varphi')\, d\varphi' \right\| = \frac{1}{t-1} < 1,$$

so the integral equation with arbitrary values $0 < t < \infty$ was solved by iteration. The first approximation for $w(\varphi)$ was taken as the constant term $Q(\varphi)$. The integral in (3) is calculated in each successive approximation for $w_k^*(\varphi)$ with an error less than $\delta_1 = \varepsilon(t-2)^2/t(t-1)$, so the following bound applies:

$$\|(w - w_k^*)\| \leqslant \frac{1}{t-2}\|(w_k^* - w_{k-1}^*)\| + \delta_1 (t-1)/(t-2).$$

The following condition was obeyed during the iterations:

$$\|(w_k^* - w_{k-1}^*)\| \leqslant \varepsilon 2 (t-2)/t$$

to ensure that the error in calculating $\|(w - w_k^*)\|$ was less than the given error $\varepsilon = 10^{-3}$ assumed in the calculations. The integral was calculated by the trapezium method with accuracy

$$\delta_1 \leqslant \frac{Mh^2(k_2 - k_1)}{6} = \frac{M(k_2 - k_1)^3}{6N^2} = \frac{\pi^3}{12(t-2)},$$

where $M = [2(t-2)]^{-1}$ is the maximum in the integrand, $K(\varphi, \varphi')$; $h = (k_2 - k_1)/N$ is the integration step, $k_1 = 0$ and $k_2 = \pi$ are the ranges in φ and $N \geq [\pi^3 t(t-1)/12\varepsilon(t-2)^3]^{1/2}$ is the number of steps in this interval. A BÉSM-4 computer was used to calculate $w(\varphi)$ for steps $t = 2.5$-30.

When $w(\varphi)$ had been found, we determined the overall flow rate of gas molecules in the space between the tubes per unit length of the cylinder l_2 with allowance for the reverse flow of particles reflected into chamber A and the flow of molecules from chamber B into A:

$$N_m = (n_a - n_b)\sqrt{\frac{R_0 T}{2\pi}}\,\frac{d}{2}\, t\mathrm{W}, \tag{4}$$

where the Clausing coefficient W is

$$\mathrm{W} = 1 - \frac{2}{t}\int_0^{\pi - \arcsin\frac{2}{t}} w(\varphi)\, Q(\varphi)\, d\varphi. \tag{5}$$

The integral in (5) was calculated by the trapezium method with an error of $0.5 \cdot 10^{-3}$.

Figure 3 shows the results for W as $W = f(t)$ (solid line), while the circles show the values of W obtained in experiments at large Knudsen numbers. There is satisfactory agreement between the experimental and theoretical values.

The Clausing coefficient with (4) gives the mass flow rate between the cylinders:

$$G_m = \frac{p_a - p_b}{\sqrt{2\pi R_0 T}} F\mathrm{W},$$

where $F = S_1 l_2 = t d l_2/2$ is the input cross section. The relation between W and the resistance of a bundle of cylinders gives us that the Euler number for gas flow between cylinders in the free-molecular state per row in the bundle is

$$\mathrm{Eu}_m = \sqrt{\frac{2\pi}{k}}\,\frac{1}{\mathrm{WM}}\left(\frac{t-2}{t}\right), \tag{6}$$

where $k = c_p/c_V$. It follows from (6) that the resistance of a cylinder bundle in the free-molecular range is dependent on the incident flow speed M and the packet geometry t.

The experimental data were represented as $Eu = f(Eu_m, Eu_c)$ for the bundles of various geometry throughout the transition range, where Eu_m and Eu_c are the Euler numbers for a bundle of tubes (cylinders) in the free-molecular and continuum regions, respectively, as referred to a single row. Figure 4 shows that when the data fall on a single relationship

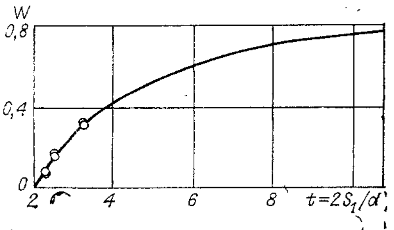

Fig. 3

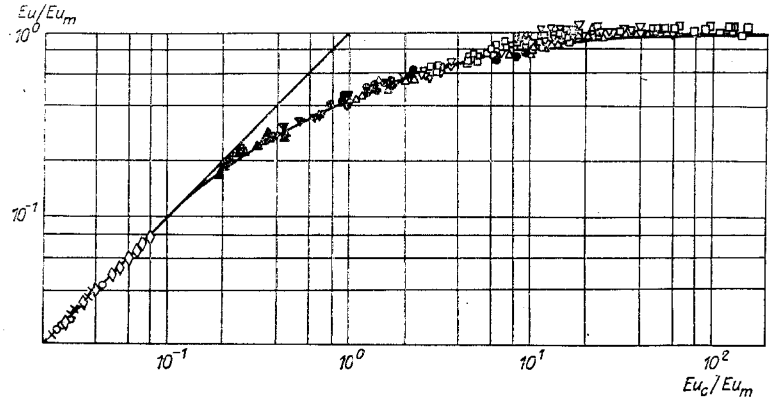

Fig. 4

asymptotic to the limiting low-density flow conditions they are independent of the bundle geometry.

The following approximating formula gives a satisfactory description of the data on chessboard bundles in subsonic flows throughout the transition region:

$$\mathrm{Eu} = \mathrm{Eu}_m[1 + 1.3(\mathrm{Eu}_m/\mathrm{Eu}_c)^{0.8}]^{-1}. \tag{7}$$

The solid line in Fig. 4 shows (7), while the experimental points correspond to the symbols in Fig. 1.

To derive the bounds to the free-molecular and continuum regions for cylinder bundles $\mathrm{Eu_c}/\mathrm{Eu_m}$ can be represented as

$$\mathrm{Eu}_c/\mathrm{Eu}_m = 21\,\mathrm{Kn_h} W\left(\frac{t}{t-2}\right)\left[1 - 0.4\,M^{1/2}(\mathrm{Kn_h})^{-1/2} + 0.2\,M^{2/3}(\mathrm{Kn_h})^{-2/3}\right]. \tag{8}$$

It follows from (8) that the limits to the flow conditions are determined to within 2% for M = 0.05-0.3 and t = 2.1-8 by the following: $\mathrm{Kn_h} \le 10^{-2}$-$1.5 \cdot 10^{-2}$ for the continuum condition and $\mathrm{Kn_h} \ge 7.8$-10.5 for the free-molecular one.

Figure 4 shows that these conditions are obeyed with $\mathrm{Eu_c}/\mathrm{Eu_m}$ of 0.13 and 160, respectively.

LITERATURE CITED

1. V. I. Gurov, E. N. Inozemtseva, et al., "The resistance of a cylinder bundle transverse to a gas flow at low Reynolds numbers," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1976).
2. Yu. A. Koshmarov and Yu. A. Ryzhov, Applied Low-Density Gas Dynamics [in Russian], Mashinostroenie, Moscow (1977).